SPARKCHARTS™

PHYSICS

SPARKCHARTS

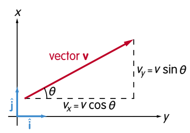
VECTORS

SCALARS AND VECTORS

- A **scalar** quantity (such as mass or energy) can be fully described by a (signed) number with units.
- A **vector** quantity (such as force or velocity) must be described by a number (its magnitude) and direction.

In this chart, vectors are bold: **v**; scalars are italicized: v.

VECTORS IN CARTE-

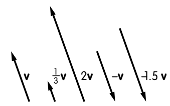


The vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the **unit vectors** (vectors of length 1) in the x-, y-, and z-directions, respectively.

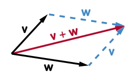
- In Cartesian coordiantes, a vector **v** can be writted as $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$, where $v_x\hat{\mathbf{i}}$, $v_y\hat{\mathbf{j}}$, and $v_z\hat{\mathbf{k}}$ are the **components** in the x-, y-, and z-directions, respectively.
- The **magnitude** (or length) of vector **v** is given by
$$v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}.$$

OPERATIONS ON VECTORS

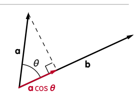
1. **Scalar multiplication**: To multiply a vector by a scalar c (a real number), stretch its length by a factor of c. The vector $-\mathbf{v}$ points in the direction opposite to **v**.



2. **Addition and subtraction:** Add vectors head to tail as in the diagram. This is sometimes called the **parallelogram method.** To subtract **v**, add $-\mathbf{v}$.



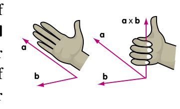
3. **Dot product** (a.k.a. **scalar product**): The dot product of two vectors gives a scalar quantity (a real number):
$$\mathbf{a}\cdot\mathbf{b} = ab\cos\theta;$$
θ is the angle between the two vectors.



- If **a** and **b** are **perpendicular,** then $\mathbf{a}\cdot\mathbf{b} = 0$.
- If **a** and **b** are **parallel,** then $|\mathbf{a}\cdot\mathbf{b}| = ab$.
- Component-wise calculation:
$$\mathbf{a}\cdot\mathbf{b} = a_xb_x + a_yb_y + a_zb_z.$$

4. **Cross product:** The cross product $\mathbf{a}\times\mathbf{b}$ of two vectors is a vector perpendicular to both of them with magnitude $|\mathbf{a}\times\mathbf{b}| = ab\sin\theta$.

- To find the direction of $\mathbf{a}\times\mathbf{b}$, use the **right-hand rule:** point the fingers of your right hand in the direction of a; curl them toward b. Your thumb points in the direction of $\mathbf{a}\times\mathbf{b}$.



Right-hand rule

- Order matters: $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$.
- If **a** and **b** are **parallel,** then $\mathbf{a}\times\mathbf{b} = 0$.
- If **a** and **b** are **perpendicular,** then $|\mathbf{a}\times\mathbf{b}| = ab$.
- Component-wise calculation:
$$\mathbf{a}\times\mathbf{b} = (a_yb_z - a_zb_y)\,\hat{\mathbf{i}} + (a_zb_x - a_xb_z)\,\hat{\mathbf{j}} + (a_xb_y - a_yb_x)\,\hat{\mathbf{k}}.$$

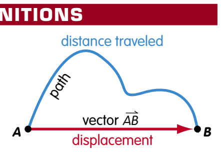
This is the determinant of the 3×3matrix
$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \end{vmatrix}.$$

KINEMATICS

Kinematics describes an object's motion.

TERMS AND DEFINITIONS

1. **Displacement** is the change in position of an object. If an object moves from position s_1 to position s_2, then the displacement is $\Delta\mathbf{s} = s_2 - s_1$. It is a vector quantity.



2. The **velocity** is the rate of change of position.
   - Average velocity: $v_{\text{avg}} = \frac{\Delta s}{\Delta t}$.
   - Instantaneous velocity: $v(t) = \lim\limits_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = \frac{ds}{dt}$.
3. The **acceleration** is the rate of change of velocity:
   - Average acceleration: $a_{\text{avg}} = \frac{\Delta v}{\Delta t}$
   - Instantaneous acceleration:
$$a(t) = \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

EQUATIONS OF MOTION: CONSTANT *a*

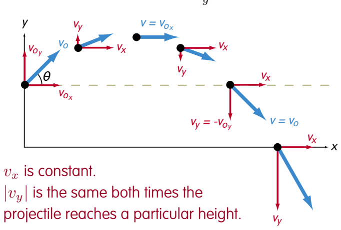
Assume that the acceleration a is constant; s_0 is initial position; v_0 is the initial velocity.

$$v_f = v_0 + at \qquad s = s_0 + v_0t + \tfrac{1}{2}at^2$$
$$v_{\text{avg}} = \tfrac{1}{2}(v_0 + v_f) \qquad = s_0 + v_ft - \tfrac{1}{2}at^2$$
$$v_f^2 = v_0^2 + 2a(s_f - s_0) \qquad = s_0 + v_{\text{avg}}t$$

PROJECTILE MOTION

A **projectile** fired with initial velocity v_0 at angle θ to the ground will trace a parabolic path. If air resistance is negligible, its acceleration is the constant **acceleration due to gravity,** $g = 9.8\,\text{m/s}^2$, directed downward.

- Horizontal component of velocity is constant:
$$v_x = v_{0x} = v_0\cos\theta.$$
- Vertical component of velocity changes:
$$v_{0y} = v\sin\theta \quad\text{and}\quad v_y = v_{0y} - gt.$$
- After time t, the projectile has traveled
$$\Delta x = v_0t\cos\theta \quad\text{and}\quad \Delta y = v_0t\sin\theta - \tfrac{1}{2}gt^2.$$
- If the projectile is fired from the ground, then the total horizontal distance traveled is $\frac{v_0^2}{g}\sin 2\theta$.

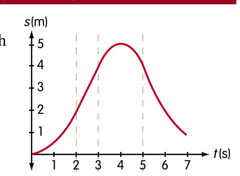


v_x is constant.
$|v_y|$ is the same both times the projectile reaches a particular height.

INTERPRETING GRAPHS

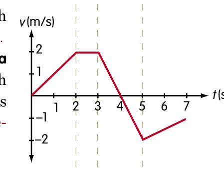
Position vs. time graph

- The **slope** of the graph gives the velocity.



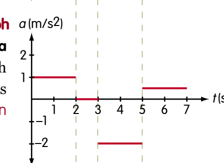
Velociy vs. time graph

- The **slope** of the graph gives the acceleration.
- The (signed) **area** between the graph and the time axis gives the displacement.



Acceleration vs. time graph

- The (signed) **area** between the graph and the time axis gives the change in velocity.



DYNAMICS

Dynamics investigates the cause of an object's motion.

- **Force** is an influence on an object that causes the object to accelerate. Force is measured in Newtons (N), where 1 N of force causes a 1-kg object to accelerate at $1\,\text{m/s}^2$.

NEWTON'S THREE LAWS

1. **First Law:** An object remains in its state of rest or motion with constant velocity unless acted upon by a net external force. (If $\sum F = 0$, then $a = 0$, and v is constant.)
2. **Second Law:** $F_{\text{net}} = ma$.
3. **Third Law:** For every action (i.e., force), there is an equal and opposite reaction ($F_{A \text{ on } B} = -F_{B \text{ on } A}$).

NORMAL FORCE AND FRICTIONAL FORCE

Normal force: The force caused by two bodies in direct contact; perpendicular to the plane of contact.

- The normal force on a mass resting on level ground is its **weight:** $F_N = mg$.
- The normal force on a mass on a plane inclined at θ to the horizontal is $F_N = mg\cos\theta$.

Frictional force: The force between two bodies in direct contact; parallel to the plane of contact and in the opposite direction of the motion of one object relative to the other.

- **Static friction:** The force of friction resisting the relative motion of two bodies at rest in respect to each other.

The maximum force of static friction is given by
$$f_{s,\,\max} = \mu_sF_N,$$
where μ_s is the **coefficient of static friction,** which depends on the two surfaces.

- **Kinetic friction:** The force of friction resisting the relative motion of two objects in motion with respect to each other. Given by $f_k = \mu_kF_N$,
where μ_k is the **coefficient of kinetic friction.**
- For any pair of surfaces, $\mu_k < \mu_s$. (It's harder to push an object from rest than it is to keep it in motion.)

FREE-BODY DIAGRAM ON INCLINED PLANE

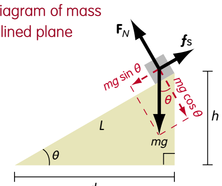
A **free-body diagram** shows all the forces acting on an object.

- In the diagram below, the three forces acting on the object at rest on the inclined plane are the force of gravity, the normal force from the plane, and the force of static friction.

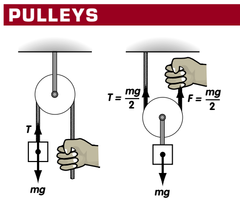
Formulas:

$$\mathbf{F}_N + \mathbf{f}_s + m\mathbf{g} = 0$$
$$F_N = mg\cos\theta$$
$$f_s = mg\sin\theta$$
$$\tan\theta = \tfrac{h}{d}$$
$$\sin\theta = \tfrac{h}{L}$$
$$\cos\theta = \tfrac{d}{L}$$

Free-body diagram of mass m on an inclined plane



PULLEYS



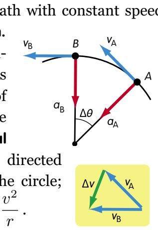
The left pulley is changing the direction of the force (pulling down is easier than up).

The right pulley is halving the amount of force necessary to lift the mass.

UNIFORM CIRCULAR MOTION

An object traveling in a circular path with constant speed experiences **uniform circular motion.**

- Even though the speed v is constant, the velocity **v** changes continually as the direction of motion changes continually. The object experiences **centripetal acceleration,** which is always directed inward toward the center of the circle; its magnitude is given by $a_c = \frac{v^2}{r}$.



- **Centripetal force** produces the centripetal acceleration; it is directed towards the center of the circle with magnitude $F_c = \frac{mv^2}{r}$.

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“WHEN WE HAVE FOUND ALL THE MEANINGS AND LOST ALL THE MYSTERIES, WE WILL BE ALONE, ON AN EMPTY SHORE.”

TOM STOPPARD

WORK, ENERGY, POWER

WORK

Work is force applied over a distance. It is measured in Joules (J): $1\,\text{N}$ of force applied over a distance of $1\,\text{m}$ accomplishes 1 J of work. ($1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ m}^2/\text{s}^2$)

- The work done by force F applied over distance s is $W = Fs$ if F and s point in the same direction. In general,
$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta,$$
where θ is the angle between $\mathbf{F}$ and $\mathbf{s}$.
- If F can vary over the distance, then $W = \int \mathbf{F}\cdot d\mathbf{s}$.

ENERGY

Energy is the ability of a system to do work. Measured in Joules.

- **Kinetic Energy** is the energy of motion, given by
$$KE = \tfrac{1}{2}mv^2.$$

- **Work-Energy Theorem:** Relates kinetic energy and work:
$$W = \Delta KE.$$
- **Potential energy** is the energy “stored” in an object by virtue of its position or circumstance, defined by
$$U_{\text{at }A} - U_{\text{at }B} = -W_{\text{from }A\text{ to }B}.$$
Ex: A rock on a hill has **gravitational potential energy** relative to the ground: it could do work if it rolled down the hill.
Ex: A compressed spring has **elastic potential energy:** it could exert a push if released. See *Oscillations and Simple Harmonic Motion: Springs.*
- **Gravitational potential energy** of mass m at height h:
$$U_g = mgh.$$
- **Mechanical energy:** The total energy is $E = KE + U$.

POWER

Power (P) is the rate of doing work. It is measured in Watts, where $1\text{ Watt} = 1\text{ J/s}$.

- Average power: $P_{\text{avg}} = \frac{\Delta W}{\Delta t}$.
- Instantaneous power: $P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$.

CONSERVATION OF ENERGY

A **conservative force** affects an object in the same way regardless of its path of travel. Most forces encountered in introductory courses (e.g., gravity) are conservative, the major exception being friction, a **non-conservative force.**

- **Conservation of energy:** If the only forces acting on a system are conservative, then the total mechanical energy is conserved: $KE_1 + U_1 = KE_2 + U_2$.

CENTER OF MASS, LINEAR MOMENTUM, IMPULSE

CENTER OF MASS

For any object or system of particles there exists a point, called the **center of mass,** which responds to external forces as if the entire mass of the system were concentrated there.

- **Disrete system:** The position vector $\mathbf{R}_{\text{cm}}$ of the center of mass of a system of particles with masses $m_1,\dots,m_n$ and position vectors $\mathbf{r}_1,\dots,\mathbf{r}_n$, respectively, satisfies
$$M\mathbf{R}_{\text{cm}} = \sum_i m_i\mathbf{r}_i\,,$$
where $M = \sum_i m_i$ is the total mass.
- **Continuous system:** If dm is a tiny bit of mass at $\mathbf{r}$, then
$$M\mathbf{R}_{\text{cm}} = \int \mathbf{r}\,dm\,,$$
where $M = \int dm$ is again the total mass.
- **Newton’s Second Law** for the center of mass:
$$\mathbf{F}_{\text{net}} = M\mathbf{A}_{\text{cm}}.$$

LINEAR MOMENTUM

Linear momentum accounts for both mass and velocity:
$$\mathbf{p} = m\mathbf{v}.$$

- For a system of particles: $\mathbf{P}_{\text{total}} = \sum_i m_i\mathbf{v}_i = M\mathbf{V}_{\text{cm}}$.
- Newton’s Second Law restated: $\mathbf{F}_{\text{avg}} = \frac{\Delta\mathbf{p}}{\Delta t}$ or $\mathbf{F} = \frac{d\mathbf{p}}{dt}$.
- Kinetic energy reexpressed: $KE = \frac{p^2}{2m}$.

Law of Conservation of Momentum

When a system experiences no net external force, there is no change in the momentum of the system.

IMPULSE

Impulse is force applied over time; it is also change in momentum.

- For a constant force, $\mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}$.
- For a force that varies over time, $\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$.

COLLISIONS

Mass m_1, moving at $\mathbf{v}_1$, collides with mass m_2, moving at $\mathbf{v}_2$. After the collision, the masses move at $\mathbf{v}_1'$ and $\mathbf{v}_2'$, respectively.

- Conservation of momentum (holds for all collisions) gives
$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'.$$
- **Elastic collisions:** Kinetic energy is also conserved:
$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1\left(v_1'\right)^2 + \tfrac{1}{2}m_2\left(v_2'\right)^2.$$
The relative velocity of the masses remains constant:
$$\mathbf{v}_2 - \mathbf{v}_1 = -\left(\mathbf{v}_2' - \mathbf{v}_1'\right).$$
- **Inelastic collisions:** Kinetic energy is not conserved. In a **perfectly inelastic collision**, the masses stick together and move at $\mathbf{v} = \mathbf{V}_{\text{cm}} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1+m_2}$ after the collision.
- **Coefficient of restitution:** $e = \frac{v_2'-v_1'}{v_1-v_2}$. For perfectly elastic collisions, $e = 1$; for perfectly inelastic collisions, $e = 0$.

ROTATIONAL DYNAMICS

Rotational motion is the motion of any system whose every particle rotates in a circular path about a common axis.

- Let $\mathbf{r}$ be the position vector from the axis of rotation to some particle (so $\mathbf{r}$ is perpendicular to the axis). Then $r = |\mathbf{r}|$ is the radius of rotation.

ROTATIONAL KINEMATICS: DEFINITIONS

Radians: A unit of angle measure. Technically unitless.
$$1\text{ revolution} = 2\pi\text{ radians} = 360°$$

Angular displacement θ: The angle swept out by rotational motion. If s is the linear displacement of the particle along the arc of rotation, then $\theta = \frac{s}{r}$.

Angular velocity ω: The rate of change of angular displacement. If v is the linear velocity of the particle tangent to the arc of rotation, then $\omega = \frac{v}{r}$.

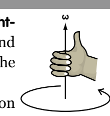
- Average angular velocity: $\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$.
- Instantaneous angular velociy: $\omega = \frac{d\theta}{dt}$.

Angular acceleration α: The rate of change of angular velocity. If a_t is the component of the particle’s linear acceleration tangent to the arc of rotation, then $\alpha = \frac{a_t}{r}$.

- Average angular velocity: $\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$.
- Instantaneous angular velociy: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$.

NOTE: The particle’s total linear acceleration $\mathbf{a}$ can be broken up into components: $\mathbf{a} = \mathbf{a}_c + \mathbf{a}_t$, where $\mathbf{a}_c$ is the **centripetal acceleration,** which does not affect the magnitude of $\mathbf{v}$, and $\mathbf{a}_t$ is the **tangential acceleration** related to α.

- **Angular velocity and acceleration as vectors:** It can be convenient to treat ω and α as vector quantities whose directions are perpendicular to the plane of rotation.
- Find the direction of $\overrightarrow{\omega}$ using the **right-hand rule:** if the fingers of the right hand curl in the direction of rotation, then the thumb points in the direction of ω.
- Equivalently, $\overrightarrow{\omega}$ points in the direction of $\mathbf{r}\times\mathbf{v}$. The equation $\overrightarrow{\omega} = \frac{\mathbf{r}\times\mathbf{v}}{r^2}$ gives both the magnitude and the direction of $\overrightarrow{\omega}$.



ROTATIONAL KINEMATICS: EQUATIONS

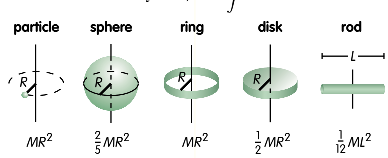
These equations hold if the angular acceleration α is constant.

$$\omega_f = \omega_0 + \alpha t \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega_{\text{avg}} = \tfrac{1}{2}(\omega_0 + \omega_f) \qquad = \theta_0 + \omega_f t - \tfrac{1}{2}\alpha t^2$$
$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_f - \theta_0) \qquad = \theta_0 + \omega_{\text{avg}}t$$

ROTATIONAL DYNAMICS

Moment of inertia is a measure of an object’s resistance to change in rotation; it is the rotational analog of mass.

- For a discrete system of masses m_i at distance r_i from the axis of rotation, the moment of inertia is
$$I = \sum_i m_i r_i^2\,.$$
- For a continuous system, $I = \int r^2\,dm$.



Torque is the rotational analog of force.

- A force $\mathbf{F}$ applied at a distance $\mathbf{r}$ from the axis produces torque
$$\tau = rF\sin\theta,$$
where θ is the angle between $\mathbf{F}$ and $\mathbf{r}$.
- Torque may be **clockwise** or **counterclockwise**. Keep track of the direction by using the vector definition of torque:
$$\overrightarrow{\tau} = \mathbf{r}\times\mathbf{F}.$$
- Analog of Newton’s second law: $\tau_{\text{net}} = I\alpha$.

Angular momentum is the rotational analog of momentum.

- A particle moving with linear momentum $\mathbf{p}$ at distance $\mathbf{r}$ away from the pivot has angular momentum
$$L = rmv\sin\theta \quad\text{and}\quad \mathbf{L} = \mathbf{r}\times\mathbf{p},$$
where θ is the angle between $\mathbf{v}$ and $\mathbf{r}$.
- For a rigid body, $\mathbf{L} = I\overrightarrow{\omega}$.
- Analog of Newton’s Second Law: $\overrightarrow{\tau}_{\text{net}} = \frac{d\mathbf{L}}{dt}$.
- **Conservation of angular momentum:** If no net external torque acts on a system, the total angular momentum of the system remains constant.

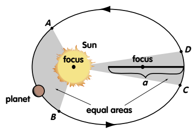
More rotational analogs:

- **Kinetic energy:** $KE_{\text{rot}} = \frac{1}{2}I\omega^2$.
The total kinetic energy of a cylindrical object of radius r rolling (without slipping) with angular velocity ω is
$$KE_{\text{tot}} = \tfrac{1}{2}m\omega^2r^2 + \tfrac{1}{2}I\omega^2.$$
- **Work:** $W = \tau\theta$ or $W = \int\tau\,d\theta$.
- **Power:** $P = \tau\omega$.

GRAVITY

KEPLER’S LAWS

1. **First Law:** Planets revolve around the Sun in elliptical paths with the Sun at one focus.
2. **Second Law:** The segment joining the planet and the Sun sweeps out equal areas in equal time intervals.
3. **Third Law:** The square of the period of revolution (T) is proportional to the cube of the orbit’s semimajoir axis a: $T^2 = \frac{4\pi^2a^3}{GM}$.



The trip from A to B takes as long as the trip from C to D.

Here a is the semimajor axis of the ellipse of revolution, M is the mass of the Sun, and $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ is the **universal gravitational constant.**

NEWTON’S LAW OF UNIVERSAL GRAVITATION

Any two objects of mass m_1 and m_2 attract each other with force
$$F = G\frac{m_1m_2}{r^2}\,,$$
where r is the distance between them (their centers of mass).

- Near the Earth, this reduces to the equation for weight: $F_W = mg$, where $g = \frac{GM_{\text{Earth}}}{R^2_{\text{Earth}}}$ is the acceleration due to gravity.

GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of mass m with respect to mass M measures the work done by gravity to bring mass m from infinitely far away to its present distance r.
$$U(r) = -\int_r^\infty \mathbf{F}\cdot d\mathbf{r} = -G\frac{Mm}{r}$$

- Near the Earth, this reduces to $U(h) = mgh$.

Escape velocity is the minimum surface speed required to completely escape the gravitational field of a planet. For a planet of mass M and radius r, it is given by
$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$

OSCILLATIONS AND SIMPLE HARMONIC MOTION

DEFINITIONS

An **oscillating system** is a system that always experiences a restoring force acting against the displacement of the system.

- **Amplitude** (A): The maximum displacement of an oscillating system from its equilibrium position.
- **Period** (T): The time it takes for a system to complete one cycle.
- **Frequency** (f or ν): The rate of oscillation, measured in Hertz (Hz), or "cycles per second." Technically, $1\,\text{Hz} = 1/\text{s}$.
- **Angular frequency** (ω): Frequency measured in "radians per second," where 2π radians $= 360°$. The unit of angular frequency is still the Hertz (because, technically, radian measure is unitless). For any oscillation, $\omega = 2\pi f$.

Period, frequency, and **angular frequency,** are related as follows:

$$T = \frac{1}{f} = \frac{2\pi}{\omega}.$$

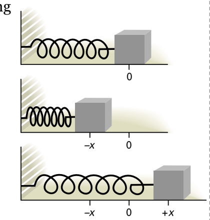
- **Simple harmonic motion** is any motion that experiences a restoring force proportional to the displacement of the system. It is described by the differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0.$$

SIMPLE HARMONIC MOTION: MASS-SPRING SYSTEM

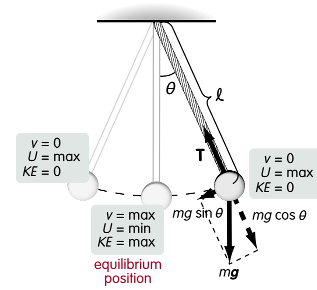
Each spring has an associated **spring constant** k, which measures how "tight" the spring is.

- **Hooke's Law:** The restoring force is given by $$F = -kx,$$ where x is the displacement from equilibrium.
- **Period:** $T = 2\pi\sqrt{\frac{m}{k}}$.
- **Frequency:** $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.
- **Elastic potential energy:** $$U = \frac{1}{2}kx^2.$$



SIMPLE HARMONIC MOTION: PENDULUM

- **Restoring force:** At angle θ, $F = mg\sin\theta$.
- **Period:** $T = 2\pi\sqrt{\frac{\ell}{g}}$.
- **Frequency:** $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$.

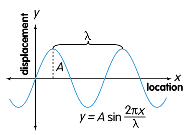


WAVES

A **wave** is a means of transmitting energy through a medium over a distance. The individual particles of the medium do not move very far, but the wave can. The direction in which the energy is transmitted is the **direction of propagation.**

DEFINITIONS

- **Transverse wave:** A type of wave where the medium oscillates in a direction perpendicular to the direction of propagation (**Ex:** pulse on a string; waves on water). A point of maxium displacement in one direction (up) is called a **crest;** in the other direction (down), a **trough.**
  - Transverse waves can either be graphed by plotting displacement versus time in a fixed location, or by plotting displacement versus location at a fixed point in time.



Displacement vs. location graph. Time is fixed.

- **Longitudinal wave:** A type of wave where the medium oscillates in the same direction as the direction of propagation (**Ex:** sound waves).
  - Longitudinal waves are graphed by plotting the density of the medium in place of the displacement. A **compression** is a point of maximum density, and corresponds to a crest. A **rarefraction** is a point of minimum density, and corresponds to a trough.

Also see definitions of **amplitude** (A), **period** (T), **frequency** (f), and **angular frequency** (ω) above.

- **Wavelength** (λ): The distance between any two successive crests or troughs.
- **Wave speed** (v): The speed of energy propagation (not the speed of the individual particles): $v = \frac{\lambda}{T} = \lambda f$.
- **Intensity:** A measure of the energy brought by the wave. Proportional to the square of the amplitude.

WAVE EQUATIONS

- Fixed location x, varying time t: $y(t) = A\sin\omega t = A\sin\left(\frac{2\pi t}{T}\right)$.
- Fixed time t, varying location x: $y(x) = A\sin\left(\frac{2\pi x}{\lambda}\right)$.
- Varying both time t and location x: $y(x,t) = A\sin\left(\omega\left(\frac{x}{v} - t\right)\right) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$.

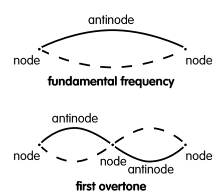
WAVE BEHAVOIR

- **Principle of Superposition:** You can calculate the displacement of a point where two waves meet by adding the displacements of the two individual waves.
- **Interference:** The interaction of two waves according to the principle of superposition.
  - **Constructive interference:** Two waves with the same period and amplitude interefere constructively when they meet **in phase** (crest meets crest, trough meets trough) and reinforce each other.
  - **Destructive interference:** Two waves with the same period and amplitude interfere destructively when they meet **out of phase** (crest meets trough) and cancel each other.
- **Reflection:** When a wave hits a barrier, it will reflect, reversing its direction and orientation (a crest reflects as a trough and vice versa). Some part of a wave will also reflect if the medium through which a wave is traveling changes from less dense to more dense.
- **Refraction:** When a wave encounters a change in medium, part or all of it will continue on in the same general direction as the original wave. The frequency is unchanged in refraction.
- **Diffraction:** The slight bending of a wave around an obstacle.

STANDING WAVES

A **standing wave** is produced by the interference of a wave and its in-synch reflections. Unlike a **traveling wave,** a standing wave does not propagate; at every location along a standing wave, the medium oscillates with a particular amplitude. Standing transverse waves can be produced on a string (**Ex:** any string instrument); standing longitudinal waves can be produced in a hollow tube (**Ex:** any woodwind instrument).

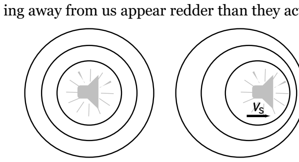
- **Node:** In a standing wave, a point that remains fixed in the equilibrium position. Caused by destructive interference.
- **Antinode:** In a standing wave, a point that oscillates with maximum amplitude. Caused by constructive interference.
- **Fundamental frequency:** The frequency of the standing wave with the longest wavelength that can be produced. Depends on the length of the string or the tube.



DOPPLER EFFECT

When the source of a wave and the observer are not stationary with respect to each other, the frequency and wavelength of the wave as perceived by the observer (f_{eff}, λ_{eff}) are different from those at the source (f, λ). This shift is called the **Doppler effect.**

- For instance, an observer moving toward a source will pass more crests per second than a stationary observer ($f_{\text{eff}} > f$); the distance between successive crests is unchanged ($\lambda_{\text{eff}} = \lambda$); the effective velocity of the wave past the observer is higher ($v_{\text{eff}} > v$).
- **Ex:** Sound: Siren sounds higher-pitched when approaching, lower-pitched when receding. Light: Galaxies moving away from us appear redder than they actually are.



Doppler effect with moving source

WAVES ON A STRING

The behavior of waves on a string depends on the force of tension F_T and the mass density $\mu = \frac{\text{mass}}{\text{length}}$ of the string.

- **Speed:** $v = \sqrt{\frac{F_T}{\mu}}$.
- **Standing waves:** A string of length L fixed can produce standing waves with $\lambda_n = \frac{2L}{n}$ and $f_n = nf_1$, where $n = 1, 2, 3, \ldots$.

SOUND WAVES

- **Loudness:** The intensity of a sound wave. Depends on the square of the amplitude of the wave.
- **Pitch:** Determined by the frequency of the wave.
- **Timbre:** The "quality" of a sound; determined by the interference of smaller waves called **overtones** with the main sound wave.
- **Beats:** Two interfering sound waves of different frequencies produce beats—cycles of constructive and destructive intereference between the two waves. The frequency of the beats is given by $f_{\text{beat}} = |f_1 - f_2|$.

DOPPLER EFFECT EQUATIONS

motion of observer	motion of source: stationary	toward observer at velocity v_s	away from observer at velocity v_s
stationary	v λ f	$v_{\text{eff}} = v$ $\lambda_{\text{eff}} = \lambda\left(\frac{v-v_s}{v}\right)$ $f_{\text{eff}} = f\left(\frac{v}{v-v_s}\right)$	$v_{\text{eff}} = v$ $\lambda_{\text{eff}} = \lambda\left(\frac{v+v_s}{v}\right)$ $f_{\text{eff}} = f\left(\frac{v}{v+v_s}\right)$
toward source at v_o	$v_{\text{eff}} = v + v_o$ $\lambda_{\text{eff}} = \lambda$ $f_{\text{eff}} = f\left(\frac{v+v_o}{v}\right)$	$v_{\text{eff}} = v \pm v_o$ $\lambda_{\text{eff}} = \lambda\left(\frac{v \pm v_s}{v}\right)$ $f_{\text{eff}} = f\left(\frac{v \pm v_o}{v \pm v_s}\right)$	
away from source at v_o	$v_{\text{eff}} = v - v_o$ $\lambda_{\text{eff}} = \lambda$ $f_{\text{eff}} = f\left(\frac{v-v_o}{v}\right)$		

CONTINUED ON OTHER SIDE

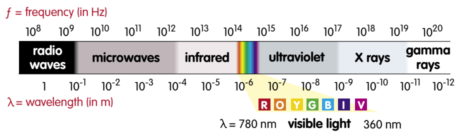
LIGHT WAVES AND OPTICS

ELECTROMAGNETIC WAVES

Light waves are a special case of transverse traveling waves called electromagnetic waves, which are produced by mutually inducing oscillations of electric and magnetic fields. Unlike other waves, they do not need a medium, and can travel in a vacuum at a speed of

$$c = 3.00 \times 10^8 \text{ m/s}.$$

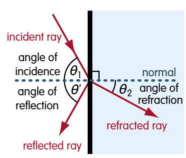
- **Electromagnetic spectrum:** Electromagnetic waves are distinguished by their frequencies (equivalently, their wavelengths). We can list all the different kinds of waves in order.
- The order of colors in the spectrum of visible light can be remembered with the mnemonic **Roy G. Biv.**



REFLECTION AND REFRACTION

At the boundary of one medium with another, part of the incident ray of light will be **reflected,** and part will be **transmitted** but **refracted.**

- All angles (of incidence, reflection, and refraction) are measured from the **normal** (perpendicular) to the boundary surface.

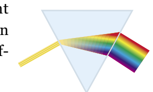


- **Law of reflection:** The angle of reflection equals the angle of incidence.
- **Index of refraction:** Ratio of the speed of light in a vacuum to the speed of light in a medium: $n = \frac{c}{v}$. In general, the denser the substance, the higher the index of refraction.
- **Snell's Law:** If a light ray travels from a medium with index of refracton n_1 at angle of incidence θ_1 into a medium with index of refraction n_2 at angle of refraction θ_2, then
$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$
- Light passing into a denser medium will bend toward the normal; into a less dense medium, away from the normal.
- **Total internal reflection:** A light ray traveling from a denser into a less dense medium ($n_1 > n_2$) will experience total internal reflection (no light is transmitted) if the angle of incidence is greater than the **critical angle,** which is given by
$$\theta_c = \arcsin\frac{n_2}{n_1}.$$

DISPERSION

Dispersion is the breaking up of visible light into its component frequencies.

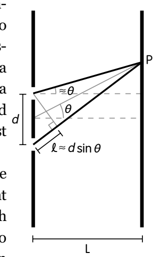
- A **prism** will disperse light because of a slight difference in refraction indices for light of different frequencies:
$$n_{\text{red}} < n_{\text{violet}}.$$



DIFFRACTION

Light bends around obstacles slightly; the smaller the aperture, the more noticeable the bending.

- **Young's double-slit experiment** demonstrates the wave-like behavior of light: If light of a single wavelength λ is allowed to pass through two small slits a distance d apart, then the image on a screen a distance L away will be a series of alternating **bright** and **dark fringes,** with the brightest fringe in the middle.



- More precisely, point P on the screen will be the center of a bright fringe if the line connecting P with the point halfway between the two slits and the horizontal make an angle of θ such that $d\sin\theta = n\lambda$, where n is any integer.
- Point P will be the center of a dark fringe if $d\sin\theta = \left(n + \frac{1}{2}\right)\lambda$, where n is again an integer.

- A **single slit** will also produce a bright/dark fringe pattern, though much less pronounced: the central band is larger and brighter; the other bands are less noticeable. The formulas for which points are bright and which are dark are the same; this time, let d be the width of the slit.

OPTICAL INSTRUMENTS: MIRRORS AND LENSES

Lenses and **curved mirrors** are designed to change the direction of light rays in predictable ways because of refraction (lenses) or reflection (mirrors).

- **Convex** mirrors and lenses bulge outward; **concave** ones, like caves, curve inward.
- **Center of curvature** (C): Center of the (approximate) sphere of which the mirror or lens surface is a slice. The radius (r) is called the **radius of curvature.**
- **Principal axis:** Imaginary line running through the center.
- **Vertex:** Intersection of principal axis with mirror or lens.
- **Focal point** (F): Rays of light running parallel to the principal axis will be reflected or refracted through the same focal point. The **focal length** (f) is the distance between the vertex and the focal point. For spherical mirrors, the focal length is half the radius of curvature: $f = \frac{r}{2}$.
- An image is **real** if light rays actually hit its location. Otherwise, the image is **virtual;** it is perceived only.

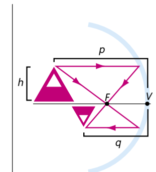
Ray tracing techniques

1. Rays running parallel to the principal axis are reflected or refracted toward or away from the focal point (toward F in concave mirrors and convex lenses; away from F in convex mirrors and concave lenses).
2. Conversely, rays running through the focus are reflected or refracted parallel to the principal axis.
3. The normal to the vertex is the principal axis. Rays running through the vertex of a lens do not bend.
4. Concave mirrors and lenses use the near focal point; convex mirrors and lenses use the far focal point.
5. Images formed in front of a mirror are real; images formed behind a mirror are virtual. Images formed in front of a lens are virtual; images formed behind are real.

LENSES AND CURVED MIRRORS

Formulas: $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\qquad \frac{\text{image size}}{\text{object size}} = -\frac{q}{p}$

Optical instrument	Focal distance f		Image distance q	Type of image
Mirror:				
Concave	positive	$p > f$	positive (same side)	real, inverted ❻
		$p < f$	negative (opposite side)	virtual, erect ❺
Convex	negative		negative (opposite side)	virtual, erect ❹
Lens:				
Convex	positive	$p > f$	positive (opposite side)	real, inverted ❸
		$p < f$	negative (same side)	virtual, erect ❷
Concave	negative		negative (same side)	virtual, erect ❶



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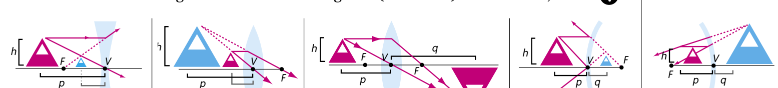

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THERMODYNAMICS

TERMS AND DEFINITIONS

Temperature measures the average molecular kinetic energy of a system or an object.

Heat is the transfer of thermal energy to a system via thermal contact with a reservoir.

Heat capacity of a substance is the heat energy required to raise the temperature of that substance by 1° Celsius.

- **Heat energy** (Q) is related to the **heat capacity** (C) by the relation $Q = C\Delta T$.

Substances exist in one of three states (**solid**, **liquid**, **gas**). When a substance is undergoing a physical change of state referred to as a **phase change:**

- Solid to liquid: **melting, fusion, liquefaction**
- Liquid to solid: **freezing, solidification**
- Liquid to gas: **vaporization**
- Gas to liquid: **condensation**
- Solid to gas (directly): **sublimation**
- Gas to solid (directly): **deposition**

Entropy (S) is a measure of the disorder of a system.

THREE METHODS OF HEAT TRANSFER

1. **Conduction:** Method of heat transfer through physical contact.
2. **Convection:** Method of heat transfer in a gas or liquid in which hot fluid rises through cooler fluid.
3. **Radiation:** Method of heat transfer that does not need a medium; the heat energy is carried in an electromagnetic wave.

LAWS OF THERMODYNAMICS

0. **Zeroth Law of Thermodynamics:** If two systems are in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
1. **First Law of Thermodynamics:** The change in the internal energy of a system U plus the work done by the system W equals the net heat Q added to the system:
$$Q = \Delta U + W.$$
2. **Second Law of Thermodynamics** (three formulations):
   1. Heat flows spontaneously from a hotter object to a cooler one, but not in the opposite direction.
   2. No machine can work with 100% efficiency: all machines generate heat, some of which is lost to the surroundings.
   3. Any system tends spontaneously towards maximum entropy.

The change in **entropy** is a reversible process defined by
$$\Delta S = \int \frac{dQ_{\text{rev}}}{T}.$$

Carnot theorem: No engine working between two heat reservoirs is more efficient than a reversible engine. The efficiency of a **Carnot engine** is given by $\varepsilon_C = 1 - \frac{T_c}{T_h}$.

GASES

Ideal gas law: $PV = nRT$, where n is the number of moles of the gas, T is the absolute temperature (in Kelvin), and $R = 8.314$ J/(mol·K) is the universal gas constant.

The ideal gas law incorporates the following gas laws (the amount of gas is constant for each one):

- **Charles' Law:** $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ if the volume is constant.
- **Boyle's Law:** $P_1V_1 = P_2V_2$ if the temperature is constant.

Translational kinetic energy for ideal gas:
$$N(KE) = N\left(\tfrac{1}{2}mv^2\right)_{\text{avg}} = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT,$$
where N is the number of molecules and $k = 1.381 \times 10^{-23}$ J/K is Boltzmann's constant.

van der Waals equation for real gases:
$$\left(P + \frac{an^2}{V^2}\right)(V - bn) = nRT$$

Here, b accounts for the correction due the volume of the molecules and a accounts for the attraction of the gas molecules to each other.

ELECTRICITY

ELECTRIC CHARGE

Electric charge is **quantized**—it only comes in whole number multiples of the **fundamental unit of charge,** e, so called because it is the absolute value of the charge of one electron.

Because the fundamental unit charge (e) is extremely small, electric charge is often measured in **Coulombs** (C). 1 C is the amount of charge that passes through a cross section of a wire in 1 s when 1 **ampere** (A) of current is flowing in the wire. (An ampere is a measure of **current**; it is a fundamental unit.)

$$e = 1.602210^{-19}\ \mathrm{C}$$

Law of conservation of charge: Charge cannot be created or destroyed in a system: the sum of all the charges is constant.

Electric charge must be **positive** or **negative.** The charge on an electron is negative.

- Two positive or two negative charges are **like** charges.
- A positive and a negative charge are **unlike** charges.

Coulomb's law: Like charges repel each other, unlike charges attract each other, and this repulsion or attraction varies inversely with the square of the distance.

- The electrical force exerted by charge q_1 on charge q_2 a distance r away is $$F_{1\text{ on }2} = k\frac{q_1 q_2}{r^2},$$ where $k = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}$ is **Couloumb's constant.**
- Similarly, q_2 exerts a force on q_1; the two forces are equal in magnitude and opposite in direcion: $$\mathbf{F}_{1\text{ on }2} = -\mathbf{F}_{2\text{ on }1}.$$
- Sometimes, Coulomb's constant is expressed as $k = \frac{1}{4\pi\varepsilon_0}$, where ε_0 is a "more fundamental" constant called the **permittivity of free space.**

ELECTRIC FIELDS

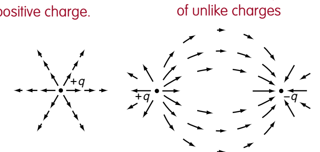
The concept of an **electric field** allows you to keep track of the strength of the electric force on a particle of any charge. If $\mathbf{F}$ is the electric force that a particle with charge q feels at a particular point, the the strength of the electric field at that point is given by $\mathbf{E} = \frac{\mathbf{F}}{q}$.

- The electric field is given in units of N/C.
- The direction of the field is always the same as the direction of the electric force experienced by a positive charge.
- Conversely, a particle of charge q at a point where the electric field has strength $\mathbf{E}$ will feel an electric force of $\mathbf{F} = \mathbf{E}q$ at that point.

Electric field due to a point charge: A charge q creates a field of strength $E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$ at distance r away. The field points towards a negative charge and away from a positive charge.

Field lines for a positive charge.

Field lines for a pair of unlike charges



The electric field is stronger when the field lines are closer together.

FLUX AND GAUSS'S LAW

Flux (Φ) measures the number and strength of field lines that go through (flow through) a particular area. The flux through an area A is the product of the area and the magnetic field perpendicular to it:

$$\Phi_E = \mathbf{E}\cdot\mathbf{A} = EA\cos\theta.$$

- The vector $\mathbf{A}$ is perpendicular to the area's surface and has magnitude equal to the area in question; θ is the angle that the field lines make with the area's surface.

Gauss's Law: The relation between the charge Q enclosed in some surface, and the corresponding electric field is given by

$$\Phi_E = \oint_s \mathbf{E}\cdot dA = \frac{Q}{\varepsilon_0},$$

where Φ_E is the flux of field lines though the surface.

ELECTRIC POTENTIAL

Just as there is a mechanical potential energy, there is an analogous **electrostatic potential energy,** which correspons to the work required to bring a system of charges from infinity to their final positions. The potential difference and energy are related to the electric field by

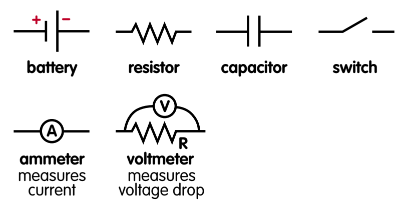
$$dV = \tfrac{dU}{q} = -\mathbf{E}\cdot d\ell.$$

The unit of potential energy is the **Volt** (V).

- This can also be expressed as $$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right).$$

ELECTRIC CURRENT AND CIRCUITS

Symbols used in circuit diagrams



Current

Current (I) is the rate of flow of electric charge through a cross-sectional area. The current is computed as $I = \frac{\Delta Q}{\Delta t}$. Current is measured in amperes, where $1\ \mathrm{A} = 1\mathrm{C/s}$.

In this chart, the direction of the current corresponds to the direction of positive charge flow, opposite the flow of electrons.

Ohm's Law: The potential difference is proportional to the current:

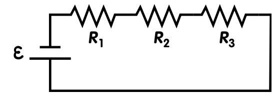
$$V = IR,$$

where R is the **resistance**, measured in **Ohms** (Ω). $1\,\Omega = 1\,\mathrm{V/A}$.

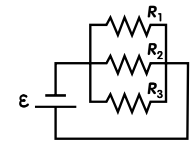
- The resistance of a wire is related to the length L and cross-sectional area A of the current carrying material by $$R = \rho\frac{L}{A},$$ where ρ is **resistivity,** which depends on the material and is measured in ohm-meters ($\Omega\cdot\mathrm{m}$).

Resistors

- **Combinations of resistors:** Multiple resistors in a circuit may be replaced by a single equivalent resistors R_{eq}.
- **Resistors in series:** $R_{\mathrm{eq}} = R_1 + R_2 + R_3 + \cdots$
- **Resistors in parallel:** $\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$



Resistors in series



Resistors in parallel

The **power** dissipated in a current-carrying segment is given by

$$P = IV = I^2R = \frac{V^2}{R}.$$

The unit for power is the **Watt** (W). $1\ \mathrm{W} = 1\ \mathrm{J/s}$.

Kirchhoff's rules

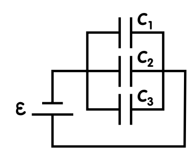
Kirchhoff's rules for circuits in steady state:

- **Loop Rule:** The total change of potential in a closed circuit is zero.
- **Junction Rule:** The total current going into a junction point in a circuit equals the total current coming out of the junction.

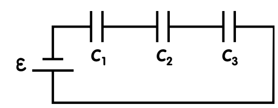
Capacitors

A **capacitor** is a pair of oppositely charged **conductors** separated by an insulator. **Capacitance** is defined as $C = \frac{Q}{V}$, where Q is the magnitude of the total charge on one conductor and V is the potential difference between the conductors. The SI unit of capacitance is the **Farad** (F), where $1\,\mathrm{F} = 1\,\mathrm{C/V}$.

- The **parallel-plate capacitor** consists of two conducting plates, each with area A, separated by a distance d. The capacitance for such a capacitor is $C = \dfrac{\varepsilon_0 A}{d}$.
- A capacitor stores electrical potential energy given by $$U = \tfrac{1}{2}CV^2.$$
- Multiple capacitors in a circuit may be replaced by a single equivalent capacitor C_{eq}.
  - **Capacitors in parallel:** $C_{\mathrm{eq}} = C_1 + C_2 + C_3 + \cdots$
  - **Capacitors in series:** $\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$



Capacitors in parallel



Capacitors in series

MAGNETISM AND ELECTROMAGNETIC INDUCTION

MAGNETIC FIELDS

A magnetic field $\mathbf{B}$ is created by a moving charge, and affects moving charges. Magnetic field strength is measured in Tesla (T), where $1\,\mathrm{T} = 1\,\mathrm{N/(A\cdot m)}$.

Magnetic force on a moving charge: A magnetic field $\mathbf{B}$ will exert a force

$$\mathbf{F} = q\,(\mathbf{v}\times\mathbf{B}),\ \text{of magnitude}$$
$$F = qvB\sin\theta$$

on a charge q moving with velocity $\mathbf{v}$ at an angle of θ to the field lines.

- Determine the direction of $\mathbf{F}$ using the **right-hand rule** (align fingers along $\mathbf{v}$, curl towards $\mathbf{B}$; the thumb points towards $\mathbf{F}$). If the charge q is negative, then $\mathbf{F}$ will point in the direction opposite to the one indicated by the right-hand rule.

Because this force is always perpendicular to the motion of the particle, it cannot change the magnitude of $\mathbf{v}$; it only affects the direction. (Much like centripetal force affects only the direction of velocity in uniform circular motion.)

- A charged particle moving in a direction parallel to the field lines experiences no magnetic force.
- A charged particle moving in a direction perpendicular to the field lines experiences a force of magnitude $F = qvB$. A uniform magnetic field will cause this particle (of mass m) to move with speed v in a circle of radius $r = \frac{mv}{qB}$.

Magnetic force on a current-carrying wire: A magnetic field $\mathbf{B}$ will exert a force

$$\mathbf{F} = I\,(\ell\times\mathbf{B}),\ \text{of magnitude}$$
$$F = I\ell B\sin\theta$$

on a wire of length ℓ carrying current I and crossed by field lines at angle θ. The direction of ℓ corresponds to the direction of the current (which in this SparkChart means the flow of positive charge).

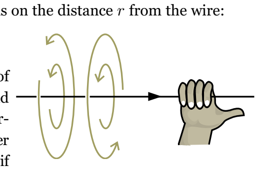
Magnetic field due to a moving charge:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\,(\mathbf{v}\times\hat{\mathbf{r}})}{r^2},$$

where μ_0 is a constant called the **permeability of free space.**

Magnetic field due to a current-carrying wire: The strength of the magnetic field created by a long wire carrying a current I depends on the distance r from the wire:

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}.$$

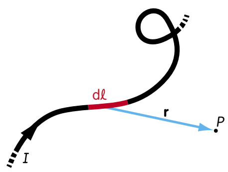


- The direction of the magnetic field lines are determined by another **right-hand rule:** if you grasp the wire with the thumb pointing in the direction of the (positive) current, then the magnetic field lines form circles in the same direction as the curl of your fingers.

MAGNETISM AND ELECTROMAGNETIC INDUCTION (continued)

Biot-Savart Law: The formula for the magnetic field due to a current-carrying wire is a simplification of a more general statement about the magnetic field contribution of a **current element** $d\vec{\ell}$. Let $d\vec{\ell}$ be a vector representing a tiny section of wire of length $d\ell$ in the direction of the (positive) current I. If P is any point in space, $\mathbf{r}$ is the vector that points from the the current element to P, and $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$ is the unit vector, then the magnetic field contribution from the current element is given by

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\left(d\vec{\ell} \times \hat{\mathbf{r}}\right)}{r^2}.$$

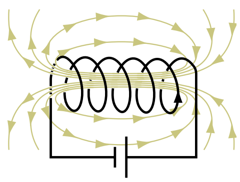


To find the total magnetic field at point P, integrate the magnetic field contributions over the length of the whole wire.

Magnetic field due to a solenoid:

$$B = \mu_0 n I,$$

where n is the number of loops in the solenoid.

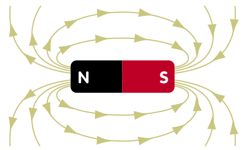


AMPERE'S LAW

Ampere's Law is the magnetic analog to Gauss's Law in electrostatics:

$$\oint_s \mathbf{B} \cdot d\ell = \mu_0 I_{\text{enclosed}}.$$

BAR MAGNETS



A **bar magnet** has a **north pole** and a **south pole.**

The magnetic field lines run from the north pole to the south pole.

ELECTROMAGNETIC INDUCTION

- Just as a changing electric field (e.g., a moving charge) creates a magnetic field, so a changing magnetic field can induce an electric current (by producing an electric field). This is **electromagnetic induction.**
- **Magnetic flux** (Φ_B) measures the flow of magnetic field, and is a concept analogous to Φ_E. See *Electricity: Flux and Gauss's Law* above. The magnetic flux through area A is
$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta.$$
Magnetic flux is measured in **Webers** (Wb), where $1\,\text{Wb} = 1\,\text{T} \cdot \text{m}^2$.

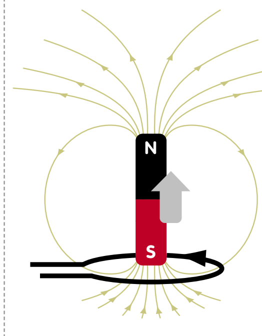
Faraday's Law: Induced emf is a measure of the change in magnetic flux over time:

$$|\varepsilon_{\text{avg}}| = \frac{\Delta\Phi_B}{\Delta t} \quad \text{or} \quad |\varepsilon| = \frac{d\Phi_B}{dt}.$$

- A metal bar rolling in a constant magnetic field $\mathbf{B}$ with velocity $\mathbf{v}$ will induce emf according to $\varepsilon = vB\ell$. The change in flux is due to a change in the area through which the magentic field lines pass.

Lenz's Law: The direction of the induced current is such that the magnetic field created by the induced current opposes the change in the magnetic field that produced it.

- Lenz's Law and Faraday's Law together make the formula
$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t} \quad \text{or} \quad \varepsilon = -\frac{d\Phi_B}{dt}.$$
- **Right-hand rule:** Point your thumb opposite the direction of the change in flux; the curl of the fingers indicated the direction of the (positive) current.
- Lenz's Law is a special case of conservation of energy: if the induced current flowed in a different direction, the magnetic field it would create would reinforce the existing flux, which would then feed back to increase the current, which, in turn would increase the flux, and so on.



As the bar magnet moves up throught the loop, the upward magnetic flux decreases.

By Lenz's law, the current induced in the loop must create more upward flux counteracting the changing magnetic field.

The induced current runs counterclockwise (looking down from the top).

An **inductor** allows magnetic energy to be stored just as electric energy is stored in a capacitor. The energy stored in an inductor is given by $U = \frac{1}{2}LI^2$. The SI unit of inductance is the **Henry** (H).

MAXWELL'S EQUATIONS

1. **Gauss's Law:** $\oint_s \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$
2. **Gauss's Law for magnetic fields:** $\oint_s \mathbf{B} \cdot d\mathbf{A} = 0$
3. **Faraday's Law:** $\oint_c \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial \Phi_B}{\partial t} = -\frac{\partial}{\partial t}\oint_s \mathbf{B} \cdot d\mathbf{A}$
4. **Ampere's Law:** $\oint_c \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}}$
5. **Ampere-Maxwell Law:**
$$\oint_c \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}} + \mu_0\varepsilon_0 \frac{\partial}{\partial t}\oint_s \mathbf{E} \cdot d\mathbf{A}$$

MODERN PHYSICS

THE ATOM

Thompson's "Raisin Pudding" model (1897): Electrons are negatively charged particles that are distributed in a positively charged medium like raisins in pudding.

Rutherford's nuclear model (1911): Mass of an atom is concentrated in the central nucleus made up of positively charged protons and neutral neutrons; the electrons orbit this nucleus in definite orbits.

- Developed after **Rutherford's gold foil experiment,** in which a thin foil of gold was bombarded with small particles. Most passed through undeflected; a small number were deflected through $180°$.

Bohr's model (1913): Electrons orbit the nucleus at certain distinct radii only. Larger radii correspond to electrons with more energy. Electrons can absorb or emit certain discrete amounts of energy and move to different orbits. An electron moving to a smaller-energy orbit will emit the difference in energy ΔE in the form of photons of light of frequency

$$f = \frac{\Delta E}{h},$$

where $h = 6.63 \times 10^{-34}\,\text{J}\cdot\text{s}$ is Planck's constant.

Quantum mechanics model: Rather than orbiting the nucleus at a specific distance, an electron is "more likely" to be found in some regions than elsewhere. It may be that the electron does not assume a specific position until it is observed. Alternatively, the electron may be viewed as a wave whose amplitude at a specific location corresponds to the probability of finding the electron there upon making an observation.

SPECIAL RELATIVITY

Postulates

1. The laws of physics are the same in all inertial reference frames. (An inertial reference frame is one that is either standing still or moving with a constant velocity.)
2. The speed of light in a vacuum is the same in all inertial reference frames:
$$c = 3.0 \times 10^8\,\text{m/s}.$$

Lorentz Transformations

If (x, y, z, t) and (x', y', z', t') are the coordinates in two inertial frames such that the the second frame is moving along the x-axis with velocity v with respect to the first frame, then

- $x = \gamma(x' + vt')$
- $y = y'$
- $z = z'$
- $t = \gamma\left(t' + \frac{x'v}{c^2}\right)$

Here, $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$.

Relativistic momentum and energy

- **Momentum:**
$$\mathbf{p} = \frac{m_0\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$
- **Energy:**
$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

PHYSICAL CONSTANTS

Acceleration due to gravity	g	$9.8\,\text{m/s}^2$
Avogadro's number	N_A	6.022×10^{23} molecules/mol
Coulomb's constant	k	$9 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$
Gravitational constant	G	$6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$
Planck's constant	h	$6.63 \times 10^{-34}\,\text{J}\cdot\text{s}$
Ideal gas constant	R	$8.314\,\text{J}/(\text{mol}\cdot\text{K})$ $= 0.082\,\text{atm}\cdot\text{L}/(\text{mol}\cdot\text{K})$
Permittivity of free space	ε_0	$8.8541 \times 10^{-12}\,\text{C}/(\text{V}\cdot\text{m})$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\text{Wb}/(\text{A}\cdot\text{m})$
Speed of sound at STP		$331\,\text{m/s}$
Speed of light in a vacuum	c	$3.00 \times 10^8\,\text{m/s}$
Electron charge	e	$1.60 \times 10^{-19}\,\text{C}$
Electron volt	eV	$1.6022 \times 10^{-19}\,\text{J}$
Atomic mass unit	u	$1.6606 \times 10^{-27}\,\text{kg}$ $= 931.5\,\text{MeV}/c^2$
Rest mass of electron	m_e	$9.11 \times 10^{-31}\,\text{kg}$ $= 0.000549\,\text{u}$ $= 0.511\,\text{MeV}/c^2$
...of proton	m_p	$1.6726 \times 10^{-27}\,\text{kg}$ $= 1.00728\,\text{u}$ $= 938.3\,\text{MeV}/c^2$
...of neutron		$1.6750 \times 10^{-27}\,\text{kg}$ $= 1.008665\,\text{u}$ $= 939.6\,\text{MeV}/c^2$
Mass of Earth		$5.976 \times 10^{24}\,\text{kg}$
Radius of Earth		$6.378 \times 10^6\,\text{m}$

SPARKCHARTS™

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